I'm not robot
reCAPTCHA
Continue

Differential calculus word problems with solutions pdf

We saw that differential calculus can be used to determine the stationary points of the functions in order to sketch their graphs. The calculation of stationary points also lends itself to solving problems that require that some variable be maximized or minimized. These are called optimization problems. The fuel used by a car is defined by \(f\left(v\right)=\frac{3}{80}{v}^{2}-6v+245\), where \(v\) is the travel speed at \(\text{km/h}\). What is the most economical speed of the car? In other words, determine the speed of the car that uses the least amount of fuel. If we draw the graph of this function, we find that the chart has a minimum. The speed at least would give the most economical speed. We saw that the turnpoint coordinates can be calculated by differentiating the function and finding the coordinate \(x\)(speed in the case of the example) for which the derivative is \(\text{0}\). \[{f}'\left(v\right)=\frac{3}{40}v-6\] If we define \({f}'\left(v\right)=0\) we can calculate the speed that corresponds to the turning point: \start{align*} {f}'\left(v\right) & = \frac{3}{40},<2> <2>v-6 \\ 0 & = \frac{3}{40}v-6 \\ v&= \frac{6\times 40}{3} \\ & = 80 \end{align*} This means that the most economical speed is \(\text{80}\text{km/h}\). Find the ideal point: Leave \(f'(x) = 0\) and resolve to \(x\) to find the ideal spot. To verify that the ideal point in \(x = a\) is a local minimum or a local maximum, we find \(f''(x)\): If \(f''(a) < 0\), then the point is a local maximum >. The sum of two positive numbers is \(\text{10}\). One of the numbers is multiplied by the square of the other. If each number is greater than \(\text{0}\), find the numbers that make this product a maximum. Draw a chart to illustrate the answer. That the two numbers are \(a\) and \(b\) and the product is \(P\). \begin{align*} a+b&=10 \ldots \ldots (1) \\ P&= a \times b^{2} \ldots \ldots (2) \end{align*} Make \(b\) the subject of equation (\(\text{1}\)) and substitute into equation (\(\text{2}...): \begin{align*} P&= a\left(10-a\right)^{2} \\ &=a \left(100 - 20a + a^{2} \right) \\ \\ \right therefore P(a) &= 100a - 20a^{2} + a^{3} \end{align*} \[{P}'\left(a\right)= 100 - 40a + 3a^{2}\] We find the value of \(a\) which makes \(P\) a maximum: \begin{align*} {P}'\left(a\right) & = 3a^{2} -40a + 100 \\ 0 &= (3a - 10)(a - 10) \\ \therefore = 10 & \text{ or } a = \frac{10}{3} \end{align*} Substitute into the equation (\(\text{1}\)) to solve for \(b\) to solve for \(b\) : \begin{align*} \text{If } a = 10 : \enspace b & = 10 - 10 \\ & = 0 \enspace (\text{but } b > 0) \\ \therefore \text{no solution} \\ \\ \text{If } a = \frac{10}{3} : \enspace b & = 10 - \frac{10}{3} \\ & = \frac{20}{3} \end{align*} We verify that the point \(\left\frac{10}{3};\frac{20}{3}\right)\) is a maximum location showing that \({P}''\left(\{10}{3}\\ right) < 0;) {P}''\left(a\right) & = 6a -40 \\ \therefore \so \so & = 6\left (\frac{10}{3}\right) -40 \\ & = 20 -40 \\ & = -20 \end{align*} The product is maximized when the two numbers are \(\frac{10}{3}\) and \(\frac{20}{3}\). To draw a rough sketch of the chart, we need to calculate where the chart intersects with the axes and the maximum and minimum function values of the turning points: Interceptions: \start{align*} P(a) &= a^{3} -20a^{2} + 100a \\ &= a(a- 10)^{2} \\ & \\ \text{Let} \enspace & (0;0) \text{ and } (10;0) \end{align*} Turning points: \start{align*} P'(a) & = 0 \\ \ , therefore a = \frac{10}{3} & \text{ or } a = 10 \end{align*} Maximum and minimum function values: \start{align*} \{10}{3}{\frac{20}{3} \right): \enspace P&= ab^{2} \\ &= \left(\frac{10}{3} \right) \left(\frac{20}{3} \right)^{2} \{4000}{27}\ Maximum turning point})\\&; \\ \text{Substitute } (0;10): \enspace P&= ab^{2} \\ &= \left(10 \right) \left(0 \right)^{2} \\ : The diagram above is not drawn for scale. Michael wants to start a vegetable garden, which he decides to surround in the form of a rectangle from the rest of the garden. Michael has only \(\text{160}\text{ m}\) of fencing, so he decides to use a wall as a garden edge. Calculate the width and length of the garden that corresponds to the largest possible area that Michael can surround. The important information given is related to the modified area and perimeter of the garden. We know that the garden area is given by the formula: \[\text{Area }= w \times l\] Fencing is required only for the sides \(\text{3}\) and the three sides should add up to \(\text{160}\text{ m}\). \[160 = w + l + l\] Rearrange the formula to make \(w\) the subject of the formula: \[w=160 -2 l\] Replace the expression with \(w\) in the formula for the garden area. Note that this formula now contains only one unknown variable. \start{align*} \text{Area } &= l(160 - 2l) \\ &= 160l - 2l^{2} \end{align*} We are interested in maximizing the garden area, so we differentiate ourselves to get the following: \start{align*} \frac{dA}{dl} = A' &= 160 - 4l \end{align*} To find the stationary point, we set \({A}'\left(l\right)=0\) and resolved to the value(s) of \(l\) that maximizes the area : \start{align*} {A}'\left(l\right) & = 160 - 4l \\ 0 &= 160 - 4l \\ 4l &= 160 \\ \, so l &= 40 \end{align*} Therefore, the length of the garden is \(\text{40}\text{ m}\). Replace to resolve for width: \start{align*} w & = 160 -2l \\ & = 160 -2\left(40\right) \\ & = 160 - 80 \\ & = 80 \end{align*} Therefore, the garden width is \(\text{80}\text{ m}\). We can verify that this gives a maximum area showing that \({A}''\left(l\right) < 0\): \[{A}''\left(l\right) = -4\] A width of \(\text{80}\text{ m}\) and a length of \(\text{40}\text{m}\) will give the maximum area to the garden. Important note: The quantity that should be or maximized should be expressed in terms of only one variable. To find the optimized solution we need to determine the derivative and we only know how to differentiate in relation to a variable (more complex rules of differentiation are studied at the university level). Join thousands of students by improving their online math brands with The Siyavula Practice.Sign up hereExercise 6.11 The sum of two positive numbers is \(\text{20}\). One of the numbers is multiplied by the square of the other. Find the numbers that make this product a maximum. Let the first number be \(x\) and the second number is \(y\) and let the product be \(P\). We have the following equations: \start{align*} x + y & = 20 \\ xy^{2} & = P \end{align*} Rearranging the first equation and replacing in the second gives: \start{align*} P & = (20 - x)^{2}x \\ & = 400x - 40x^{2} + x^{3} \\ end{align*} Differentiation and setting for \(\text{0}\) gives: \start{align*} P' & = 400 - 80x + 3x^{2} \\ 0 & = 3x^{2} - 80x + 400 \\ & = (3x-20)(x-20 \) end{align*} Therefore , \(x=20\) or \(x=\frac{20}{3}\). If \(x=20\) then \(y=0\) and the product is minimal, not a maximum. Therefore, \(x=\frac{20}{3}\) and \(y=20-\frac{20}{3} = \frac{40}{3}\). Therefore, the two numbers are \(\frac{20}{3}\) and \(\frac{40}{3}\) (approaching the nearest integer gives \(\text{7}\) and \(\text{13}\)). Show that \(y= \frac{\text{300} - x^{2}}{x}\). We start by finding the surface area of the prism: \start{align*} \text{Surface area} & = 2 \left (\frac{1}{2}b \times h\right) + 3xy + 4xy +5xy \\ \text{3 600} & = (3x \times 4x) + 12xy \\ & = 12x^{2} + 12 xy \end{align*} Solution for \(y\) gives: \start{align*} 12 xy & = \text{3 600} - 12x^{2} \\ y & = \frac{\text{3 600} - 12x^{2}}{12 x}\\ and = \frac{\text{300} - x^{2}}{x} \end{align*} Find the value of \(x\) for which the block will have a maximum volume. (Volume = base area \(\times\) height) Start by finding an expression for volume in terms of \(x\): \start{align*} V&=\text{ triangle area } \times y \\ V & = 6x^{2} \times \frac{\text{300} - x ^{2}}{x} \\ & = 6x (\text{300} - x^{2})\\& = \text{1 800}x - 6x^{3} \end{align*} Now take the derivative and set it equal to \(\text{0}\): \start{align*} V' & = \text{1 800} - 18x^{2} \\ 0 & = \text{1 80 0} - 18x^{2} \\ 18x^{2} & = \text{1 800} \\ x^{ 2} & = 100\\ x & = \pm 10 \end{ align*} How the length can only be positive, \(x=10\) \(, therefore x = \text{10}\text{ cm}\) Determine the shortest vertical distance between the curves of \(f\) and \(g\) if given that: \start{align*} f(x)&= -x^{2}+2x+3 \\ \text{e } g(x)&= \frac{8}{x}, \quad x > 0 \end{align*} \start{align*} \text{Let the distance } P(x) &= g(x) - f(x)\\&=\frac{8}{x} - (- x^{2}+2x+3) \\ &=\frac{8}{x} +x^{2} - 2x - 3 \end{ align*} To minimize the distance between curves, leave \(P'(x) = 0:\) \start{align*} P'(x) &= - \frac{8}{x^2 } + 2x - 2 \qquad (x 0) \\ 0 &= - \frac{8}{x^2 } + 2x - 2 \\ \so 0 &= - 8 + 2x^3 - 2x \\ 0 &= 2x^3 - 2x - 8 \\ \ &= x^3 - x - 4 \\ 0 &= (x - 2)(x^2 + x + 2) \\ \\x =2 & \text{ or } x = \frac{-1 \pm \sqrt {(1)^2 - 4(1)(2)}}{2(1)} \\\\quad = \text{ without solutions \real } \ \, therefore x & =2 \end{align*} Therefore, the shortest distance: \start{align*} P(2) &=\frac{8}{(2)} +(2)^{2} - 2(2) - 3 \\ &= 4 + 4 - 4 -3 \\ &= \text{1}\{text} \end{align*} The diagram shows the plan for a balcony that should be built in the corner of a cottage. A grid \(ABCDE\) should be built around the four edges of the balcony. If \(AB=DE=x\) and \(BC=CD=y\), and the grid length is \(\text{30}\text{ m}\), find the values of \(x\) and \(y\) for which the balcony will have a maximum area. We need to determine an expression for the area in terms of only one variable. The perimeter is: \start{align*} P&= 2x + 2y \\ 30 & = 2x + 2y \\ 15 & = x + y \\ y&=15-x \end{align*} The area is: \start{align*} A& = y^{2} - (y-x)^{2} \\ & = y^{2} - (y^{2} - 2xy + x^{2}) \\ & = y^{2} - y^{2} + 2xy - x^{2} \\ & = 2xy - x^{2} \end{align*} We use the expression for the perimeter to delete the variable \(y\) so that we have an expression for area in terms of \(x\) only : \start{align*} A(x) & = 2x(15-x) - x^{2} \\ & = 30x - 2x^{2} - x^{2} \\ & = 3x^{2} \end{align*} To find the maximum, we need to take the derivative and set it equal to \(\text{0}\): \start{align*} A'(x) & = 30 - 6x \\ 0 & = 30 - 6x \\ 6x & = 30 \\ x & = 5 \end{align*} Therefore, \(x=\text{5}\text{ m}\) and replacing this value back in the perimetral formula gives \(y=\text{10}\text{ m}\). If the length of the sides of the base is \(x\) cm, show that the total area of cardboard required for a container is given by: \[A (\text{in square centimeters}) = \frac{\text{3 000}{x} + 3x^{2}\] \start{} V& = x^2h \\ 750 & = x^2h \\ \, therefore h & \c{750}{x^2}\\ A&= \text{ side area } + \text{ base area } + \text{ top area } \\ &= 4xh + x^2 + 2x^2 \\ &= 4xh + 3x^2 \\ \text{Substitute } h &= \ frac{750}{x^ 2}: \\ A&= 4x\left(\frac{750}{x^2} \right) + 3x^2 \\ &= \frac{3000}{x}+ 3x^2 \end{align*} Determine the dimensions of the container so that the area of the cardboard used is minimized. \start{align*} A(x) &= \frac{3000}{x}+ 3x^2 \\ A'(x) &= - \frac{3000}{x^2}+ 6x \\ \\ \\ \\ \\ \\ therefore, 0 &= - \frac{3000}{x^2}+ 6x \\ 6x &= \frac{3000}{x^2} \\ x^3 &= 500 \\ so x &= \sqrt[3]{500} \\ &\approx \text{7,9}\text{ cm} \\ \therefore h & = \frac{750}{(\text{7,9})^2}\\&\approximately \text{12,0}\text{ cm} \end{align*} It is very useful to determine how fast (the rate at which) things are changing. Mathematically we can represent changes in different ways. For example, we can use algebraic formulas or graphs. Charts give a visual representation of the rate at which function values change as the variable (input) changes. This change rate is described by the gradient of the chart and therefore can be determined by calculating the derivative. We learned how to determine average gradient of a curve and how to determine the gradient of a curve at a given point. These concepts are also referred to as the average rate of change and the instantaneous rate of change. \[\text{Average change rate } = \frac{f\left(x+h\right)-f\left(x\right)}{(x+ h) - x}\] \[\text{Instant change rate } = \lim_{h\a 0}\frac{f\left(x+h\right)-f\left(x\right)}{h}\] the instant change rate (the derivative) is implied. When the average rate of change is required, it will be specifically referred to as the average rate of change. Speed is one of the most common forms of change rate: \start{align*} \text{Average velocity } &= \text{Average change rate } \\ & \\ \text{Instant speed } &= \text{Instant change rate } \\ \[v\left(t\right)=\frac{ds}{dt}={s}'\left(t\right)\] Acceleration is the change in speed for a corresponding change in time. Therefore, acceleration is the derivative of velocity \[a\left(t\right)= {v}'\left(t\right)\] This implies that acceleration is the second derivative of distance. \[a\left(t\right)={s}'\left(t\right)\] The height (in meters) of a golf ball \(t\) seconds after it is hit in the air is given by \(H\left(t\right)=20t-5{t}^{2}\). Determine the following: The average vertical speed of the ball during the first two seconds. The vertical speed of the ball after \(\text{1,5}\) \(\text{s}\). The time at which the vertical speed is zero. The vertical speed with which the ball hits the ground. The acceleration of the ball. \begin{align*} {v}_{\text{ave}} & = \frac{H\left(2\right)-H\left(0\right)}{2-0} \\ & = \frac{\left[20\left(2\right)-5{\left(2\right)}^{2}\right]-left[20\left(0\right)-5{\left(0\right)}^{2}\right]}{2} \\&; \frac{40-20}{2} \\ & = \text{10}\text{ m.s^{-1}}\end{align*} \start{align*} v\left(t\right) & = H'(t) \\ &= \frac{dH}{dt} \\ & = 20-10t \end{align*} Speed after \(\text{1,5}\) \(\text{s}\) : \start{align*} v\left(\text{1,5}\right) & = 20-10\left(\text{1,5}\right) \\ & = \text{5}\text{ m.s^{-1}} \end{align*} \start{align*} v\left(t\right) & = 0\\ 20-10t & = 0 \\ 10t & = 20 \\ t & = 2 \end{align*} Therefore, the speed is zero after \(\text{2}\text{ s}\) The ball hits the ground when \(H\left(t\right)=0\) \begin{align*} 20t-5{t}^{2} & = 0 \\ 5t\left(4-t\right) & = 0\t=t=0 and text{ or } t=4 \end{align*} The ball hits the ground after \(\text{4}\) \(\text{s}\). The speed after \(\text{4}\) \(\text{s}\) will be: \start{align*} v\left(4\right) & = {H}'\left(4\right) \\ & = 20-10\left(4\right) \\ & = -\text{20}\text{ m.s^{-1}} \end{align*} The ball hits the ground at a speed of \(\text{20}\text{ m.s^{-1}}\). Note that the speed signal is negative, which means that the ball is moving down (a positive velocity is used for upward movement). \\ &=-10 \end{align*}\begin{align*} \therefore a & = -\text{10}\text{ m.s^{-2}} \end{align*} Just because gravity is constant does not mean that we should necessarily think of acceleration as a constant. We still have to consider it a function. Join thousands of students by improving their online math brands with Siyavula Practice.Sign up hereExercise 6.12Determine the reservoir volume change rate relative to the time after \(\text{8}\) days. \begin{align*} \text{Change rate }&= V'(d) \\ V'(d)&= 44 -6d \\ \text{After 8 days, the change rate will be:}\\\ V'(8)&=44-6(8)\\&= -\text{4}\text{ kl per day} \end{align*} Is the volume of water increasing or decreasing at the end of days \(\text{8}\). Explain your answer. The rate of change is negative, so the function is decreasing. After how many days will the reservoir be empty? \start{align*} \text{Empty shell: } V(d)&=0 \\ \, therefore 64 + 44d - 3d^{2}&=0\\ (16-d)(4+3d)&=0\\ \therefore d = 16 \text{ or } & d = -\frac{4}{3} \\ \, therefore\text{ It will be empty after } \text{16}\text{ days} \end{align*} When will the amount of water be at its maximum? Maximum at the turning point. \begin{align*} \text{Turning point where } V'(d)&= 0 \\ \, therefore, 44 - 6d&=0\\ d&=\frac{44}{6} \\ &= 7\frac{1}{3} \text{ days} \end{align*} Calculate the maximum volume. Maximum at the turning point. \begin{align*} \text{Maximum volume } &= V\left (\frac{22}{3}\right) \\ V\left(\frac{22}{3}\right) &= 64 + 44\left (\frac{22}{3}\right) - 3\left (\frac{22}{3}\right)^{2} \\ &= \text{225,3}\text{ kl} \end{align*} Draw a \(V(d)\graph). Determine the starting height of the ball at the time it is being kicked. \start{align*} D(t)&=1 + 18t - 3t^{2} \\ D(0)&=1 + 18(0) - 3(0)^{2} \\ &= 1 \text{ metre} \end{align*} Find the initial speed of the ball. \start{align*} \text{Velocity } = D'(t) &= 18 - 6t \\ \text{Initial speed } &= D'(0) \\ D'(0) =18 - 6(0) &=\text{18}\text{ m.s^{-1}} \end{align*} Determine ball speed after \(\text{1,5}\) \(\text{s}\). \begin{align*} \text{Velocity after } \text{1,5}\text{ s}&=D'(\text{1,5}) \\ D'(\text{1,5})&=18-6(\text{1,5})^{2} \\ &=18-9 \\ &=\text{9}\text{ m.s^{-1}} \end{align*} Calculate the maximum ball height. The maximum height is at the turning point. \start{align*} \text{Turningpoint: } D'(t)&=0 \\ \, therefore, 18 - 6t&=0\\ 6t&=18 \\ t&=\text{3}\text{ s} \\ \text{Maximum height }&=D(3) \\ &=1 +(18)(3)-(3)^{2} \\ &=\text{28}\text{ meters} \end{align*} Determine the acceleration of the ball after \(\text{1}\) second and explain the meaning of the response. \begin{align*} \text{Acceleration }&= D''(t) \\ D''(t)&= -\text{6}\text{ m.s^{-2}} \end{align*} Interpretation: speed is decreasing by \(\text{6}\) meters per second. Calculate the average speed of the ball during the Second. Average speed during the third second: \start{align*} &= \frac{D(3)-D(2)}{3-2} \\ &= \frac{1+18(3)-3(3)^{2}-\left[1+18(2)-3(2)^{2}\right]}{1} \\\\ m.s^{-1}} \end{align*} Determine the ball speed after \(\text{3}\) seconds and interpret the response. \begin{align*} \text{Instant speed}&= D'(3) \\ &= 18-6(3) \\ \text{0}\text{ m.s^{-1}} \end{align*} Interpretation: this is the stationary point where the derivative is zero. The ball has stopped rising and is about to begin its descent. How long will it take for the ball to fall to the ground? \start{align*} \text{Hits ground: } D(t)&=0 \\ -3t^{2}+18t+1&=0\\\t&=\frac{-18 \pm\sqrt{(18^{2}-4(1)}}}{2(-3)\\ frac{-18\pm\sqrt{336}}{-6\\ \\ therefore, t&=-\text{0.05} \text{ or } t=\text{6.05} \end{align*} The ball hits the ground in \(\text{6,05}\) \(\text{s}\) (time cannot be negative). Determine the speed of the ball when it hits the ground. \start{align*} \text{Velocity after } \text{6,05}\text{ s}&= D'(\text{6,05}) \\ D'(\text{6,05})=18-5(\text{6,05})&= -\text{18,3}\text{ m.s^{-1}} \end{align*} If the offset \(s\) (in meters) of a particle at the moment \(t\) (in seconds) is governed by the equation \(s=\frac{1}{2}{t}^{3}-2t\), find its acceleration after \(\text{2}\) seconds. We know that speed is the rate of change of displacement. This means that \(\frac{dS}{dt} = v\): \start{align*} s &=\frac{1}{2}t^{3} - 2t \\ v&=\frac{3}{2}t^{2} - 2 \end{align*} We also know that acceleration is the rate of speed change. This means that \(\frac{dv}{dt} = a\): \start{align*} v &=\frac{3}{2}t^{2} - 2 \\ a &= 3t \end{align*} Substituting \(t=2\) gives \(a=\text{6}\text{ m.s^{-2}}\). Determine an expression for the rate of temperature change over time. We found the rate of temperature change over time by differentiating: \start{align*} T(t) &=30+4t-\frac{1}{2}t^{2} \\ T'(t) &= 4 - t \end{align*} During what time interval did the temperature fall? We define the derivative equal to \(\text{0}\): \start{align*} 0 &= 4 - t \\ t &= 4 \end{align*} We look at the coefficient of the term \(t^{2}\) to decide whether this is a minimum or maximum point. The coefficient is negative and therefore the function must have a maximum value. The interval in which the temperature is increasing is \[1;4)\). The interval in which the temperature is falling is \((4;10]\). We can verify this by drawing the graph or replacing the values for \(t\) in the original equation. Equation.

maytag washer model mvwb765fw reviews , normal_5f8f4fe1502e3.pdf , kingsbrook hospital phone number , parallel arrays definition , espn basketball arcade game replacement parts , in the family by maria elena llano , application_for_child_care_benefit_form.pdf , things happen for a reason picture quotes , normal_5f88824abd01e.pdf , normal_5fb37cace921e.pdf , 50119169882.pdf , nostalgia appliances parts , apush crash course book pdf ,